Quantisation Issues in Feedback Control

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I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Hernan Haimovich

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Ruth for sharing so much with me: playing music and dancing together, making me feel at home at their place, going on holidays together, and so much more. Thanks a lot, guys.

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To Silvana

Contents

Abstract

Systems involving quantisation arise in many areas of engineering, especially when digital implementations are involved. In this thesis we consider different aspects of quantisation in feedback control systems. We study two topics of interest: (a) quantisers that quadratically stabilise a given system and are efficient in the use of their quantisation levels and (b) the derivation of ultimate bounds for perturbed systems, especially when the perturbations arise from the use of quantisers.

In the first part of the thesis we address problem (a) above. We consider quadratic stabilisation of discrete-time multiple-input systems by means of quantised static feedback and we measure the efficiency of a quantiser via the concept of quantisation density. Intuitively, the lower the density of a quantiser is, the more separated its quantisation levels are. We thus deal with the problem of optimising density over all quantisers that quadratically stabilise a given system with respect to a given control Lyapunov function. Most of the available results on this problem treat single-input systems, and the ones that deal with the multiple-input case consider only two-input systems. In this thesis, we derive several new results for multiple-input systems and also provide an alternative approach to deal with the single-input case. Our new results for multiple-input systems include the derivation of the structure of optimal quantisers and the explicit design of multivariable quantisers with finite density that are able to quadratically stabilise systems having an arbitrary number of inputs. For single-input systems, we provide an alternative approach to the analysis and design of optimal quantisers by establishing a link between the separation of the quantisation levels of a quantiser and the size of its quantisation regions.

In the second part of the thesis we address problem (b) above. In the presence of perturbations, asymptotic stabilisation may not be possible. However, there may exist a bounded region that contains the equilibrium point and has the property that the system trajectories converge to this bounded region. When this bounded region exists, we say that the system trajectories are ultimately bounded, and that this bounded region is an ultimate bound for the system. The size of the ultimate bound quantifies the performance of the system in steady state. Hence, it is important to derive ultimate bounds that are as tight as possible. This part of the thesis addresses the problem of ultimate bound computation in settings involving several scalar quantisers, each having different features. We consider each quantised variable in the system to be a perturbed copy of the corresponding unquantised variable. This turns the original

quantised system into a perturbed system, where the perturbation has a natural *componentwise* bound. Moreover, according to the type of quantiser employed, the perturbation bound may depend on the system state. Typical methods to estimate ultimate bounds are based on the use of Lyapunov functions and usually require a bound on the norm of the perturbation. Applying these methods in the setting considered here may disregard important information on the structure of the perturbation bound. We therefore derive ultimate bounds on the system states that explicitly take account of the componentwise structure of the perturbation bound. The ultimate bounds derived also have a componentwise form, and can be systematically computed without having to, for example, select a suitable Lyapunov function for the system. The results of this part of the thesis, though motivated by quantised systems, apply to more general perturbations, not necessarily arising from quantisation.

Notation

$\triangleq$	Definition.
$\lVert\cdot\rVert_2$	Euclidean norm of a vector and corresponding induced norm of a matrix.
$\lVert\cdot\rVert_\infty$	Infinity norm of a vector and corresponding induced norm of a matrix.
$\uplus$	Disjoint union.
$\#$	Number of elements (cardinality) of a set.
$\eta(q)$	Quantisation density of a quantiser q.
$\lambda_{\max}(\cdot)$	Maximum eigenvalue of a real symmetric matrix.
$\lambda_{\min}(\cdot)$	Minimum eigenvalue of a real symmetric matrix.
$\rho(M)$	Spectral radius of the square matrix M, that is, maximum over the magnitude of the eigenvalues of M.
$\mathbf{0}_{n\times m}$	$n \times m$ matrix with zero entries.
$\mathbf{1}_n$	n-dimensional column vector with all components equal to 1.
$\lfloor b \rfloor$	Greatest integer not greater than b.
$\lceil b \rceil$	Least integer not less than b.
$\mathbb{C}$	Set of complex numbers.
CAQS	Coarse Almost Quadratically Stabilising.
CLF	Control Lyapunov Function.
$\mathrm{diag}(\lambda_1,\ldots,\lambda_n)$	Diagonal matrix with main-diagonal entries $\lambda_1,\ldots,\lambda_n$.
I_n	$n \times n$ identity matrix.
$\mathrm{Im}(M)$	Range of a matrix M.
LTI	Linear Time-invariant.
$\max\{x,y\}$	Componentwise maximum of vectors x and y.
$\lvert M \rvert$	Elementwise magnitude of a matrix M with (possibly) complex entries. That is, if M has entries $M_{i,j}$, then $\lvert M \rvert$ is the matrix with entries $\lvert M_{i,j} \rvert$.

$M \prec N$	Set of componentwise inequalities $M_{i,j} < N_{i,j}$ for $i = 1, \ldots, m$, $j = 1, \ldots, n$, when $M, N \in \mathbb{R}^{m \times n}$.
$M \preceq N$	Set of componentwise inequalities $M_{i,j} \leq N_{i,j}$, for $i = 1, \ldots, m$, $j = 1, \ldots, n$, when $M, N \in \mathbb{R}^{m \times n}$.
$M \succ N$	Set of componentwise inequalities $M_{i,j} > N_{i,j}$ for $i = 1, \ldots, m$, $j = 1, \ldots, n$, when $M, N \in \mathbb{R}^{m \times n}$.
$M \succeq N$	Set of componentwise inequalities $M_{i,j} \geq N_{i,j}$, for $i = 1, \ldots, m$, $j = 1, \ldots, n$, when $M, N \in \mathbb{R}^{m \times n}$.
$n \bmod N$	Remainder of dividing n by N ($n \in \mathbb{Z}_{+,0}$, $N \in \mathbb{Z}_+$).
QS	Quadratically Stabilising.
$q, \tilde{q}, \bar{q}, \mathring{q}$	Quantisers.
$\mathbb{R}$	Set of real numbers.
$\mathbb{R}^n_+$	Set of vectors in $\mathbb{R}^n$ with positive components.
$\mathbb{R}^n_{+,0}$	Set of vectors in $\mathbb{R}^n$ with nonnegative components.
$\mathbb{R}^{n \times m}_+$	Set of matrices in $\mathbb{R}^{n \times m}$ with positive entries.
$\mathbb{R}^{n \times m}_{+,0}$	Set of matrices in $\mathbb{R}^{n \times m}$ with nonnegative entries.
$\mathbb{Re}(M)$	Elementwise real part of a matrix M. That is, if M has entries $M_{i,j}$, then $\mathbb{Re}(M)$ is the matrix with entries $\mathbb{Re}(M_{i,j})$.
^{T}	Transpose.
T^k	Iteration of a map $T : \mathbb{R}^n \to \mathbb{R}^n$, $T^k(x) = T(T^{k-1}(x))$, for $k = 1, 2, \ldots$, and $T^0(x) \triangleq x$.
$\mathcal{U}(q)$	Range of a quantiser q.
$V(\cdot)$	Control Lyapunov function.
$\mathbb{Z}$	Integers.
$\mathbb{Z}_+$	Positive integers.
$\mathbb{Z}_{+,0}$	Nonnegative integers.

Chapter 1

Introduction

1.1 Quantisation in Feedback Control

The term "quantisation" refers to the restriction of a variable to a discrete set of values rather than a continuous set of values. There are several reasons why quantisation needs to be considered in feedback control systems. For example, since controllers are usually implemented digitally, signals that take values in a continuous set need to be represented with finite precision to allow digital information processing in finite time (Åström and Wittenmark, 1997). In addition, sensors may produce an output that indicates only whether the value of the measured signal lies within some range. The number of different ranges that the sensor can distinguish may be severely limited in some cases. A prime example of the latter situation is provided by the exhaust gas oxygen sensor used for air-to-fuel ratio control in automotive systems (see, for example, Grizzle et al., 1991). Further motivation for considering quantisation in feedback control systems is the recent boom of interest in networked control systems (Raji, 1994; Zhang et al., 2001; Walsh and Ye, 2001). These systems are characterised by the fact that controllers and plants are interconnected over a digital communication network, making quantisation essential in order to transmit information among different parts of the system.

The analysis of the effects of quantisation in feedback control systems began as early as in the 1950s, as evidenced by the work of Kalman (1956). Kalman's work was aimed at studying the effects of nonlinearities on sampled-data systems. Although the words *quantiser* or *quantisation* do not explicitly appear in this work, the effect of including an ideal relay —one of the simplest forms of quantiser— in a sampled-data system was analysed. Also during the 1950s, the words *quantisation* and *quantised* appeared in the control systems literature (Flügge-Lotz and Taylor, 1956; Bertram, 1958). The need to analyse the effects of quantisation on control systems stemmed from the fact that controllers could be digitally implemented. Indeed, in a typical digital control scheme, quantisation arises due to the use of analog-to-digital (A/D) and digital-to-analog (D/A) converters, and because the calculations performed

by the digital processor have round-off errors.

The way in which quantisation is dealt with in feedback control systems has experienced a striking change in recent years. Traditionally, quantisation in control systems was regarded as an undesirable phenomenon. The most common approach to designing a digital controller was to disregard quantisation in a first stage, when the controller was designed. The impact of quantisation on the resulting performance was later mitigated by utilising A/D and D/A converters, and digital processors, with suitably high precision. Naturally, however, even small quantisation errors can have a negative impact on achievable performance. Different methods exist for estimating the deleterious effect of quantisation on a digital control system that is designed ignoring the presence of quantisation (Bertram, 1958; Slaughter, 1964; Yakowitz and Parker, 1973; Green and Turner, 1988; Miller et al., 1988; Farrell and Michel, 1989; Miller et al., 1989). Almost all of these methods regard a quantised variable as a perturbed copy of the unquantised variable. The effect of quantisation on the resulting performance is then analysed utilising a bound on the perturbation introduced by the quantiser. This approach is well justified when the required precision is easily achievable and the cost of the resulting implementation is reasonable.

A different approach is to regard a quantised variable as a partial observation of the unquantised variable. This means that a quantised variable provides information on a range of values that the unquantised variable may take, rather than one specific value. Relevant works that address quantisation in this manner in the context of feedback control systems are Curry (1970) and Delchamps (1990). These works have paved the way for the most recent approach to quantisation, which consists in viewing a quantiser as an *information coder*. This new approach has led to a paradigm change regarding quantisation in a feedback control system: from undesirable phenomenon to intrinsic and inescapable system component.

Indeed, in recent years, different control schemes have been proposed and analysed, which explicitly account for the fact that controller and plant(s) are connected via a communication channel (see the special issue Antsaklis and Baillieul, Guest Eds., 2004, and the references therein). The new challenges that arise from the introduction of a communication channel between controller and plant(s) are numerous. These challenges include the need to explicitly deal with variable time delays, nonuniform sampling, limited data-rate/bandwidth, data loss, and quantisation.

There has been substantial research effort directed at various aspects of the above factors. Several lines of research exist which address different groups of such issues. In particular, numerous works explicitly deal with quantisation while focusing on stabilisation in a networked control setting. Within these works, we can distinguish between the ones where the quantisation strategy is dynamic and time-varying (for example, Wong and Brockett, 1999; Brockett and Liberzon, 2000; Liberzon, 2003a,b; Liberzon and Hespanha, 2005; Nair and Evans, 2003, 2004; Li and Baillieul, 2004; Tatikonda and Mitter, 2004a,b; Tatikonda and Elia, 2004) and where it is fixed and static (for example, Elia and Mitter, 2001; Elia and Frazzoli, 2002; Kao and Venkatesh, 2002; Fu and Xie, 2003, 2005; Baillieul, 2002; Ishii

and Francis, 2002b, 2003; Ishii and Başar, 2005; Goodwin et al., 2004).

If the quantisation strategy is dynamic and time-varying, then quantisers having a finite number of levels may be employed to achieve asymptotic stabilisation. On the other hand, if the quantisation strategy is fixed and static, employing a quantiser with a finite number of levels can only yield local practical stability. Asymptotic stability can be achieved by utilising quantisers with a countably infinite number of levels, having increasingly higher precision towards the origin (such as logarithmic quantisers). Quantisers with a finite number of levels have greater practical significance than quantisers with an infinite number of levels. However, the latter quantisers have been very useful for proving many important results in networked control.

Throughout this thesis, we will regard a quantiser as a fixed and static component of the system and will analyse different aspects of quantisation in feedback control systems. Specifically, we will deal with the following two topics: (a) quadratic stabilisation by means of quantised static feedback and (b) the derivation of ultimate bounds for perturbed systems, especially when the perturbations arise due to the use of quantisers.

1.1.1   Quadratic Stabilisation via Quantised Static Feedback

In Part I of the thesis, we deal with quadratic stabilisation of discrete-time linear systems by means of static feedback employing quantisers. The approach that we follow is related to the work of Elia and Mitter (2001); Elia and Frazzoli (2002); Elia (2002); Kao and Venkatesh (2002); Fu and Xie (2003, 2005). Elia and Mitter (2001) introduce a measure of density of quantisation. Intuitively, the density of a quantiser is lower than that of another quantiser if the values of the former are more separated than those of the latter. In this sense, a quantiser can be regarded as being more efficient in the use of its quantisation levels if its density is lower. In this context, an important question that is posed and answered in Elia and Mitter (2001) is: for a linear single-input system, what is the most efficient quantiser over all quadratically stabilising quantisers?

The interesting results of Elia and Mitter (2001) apply only to single-input systems. Generalising these results to multiple-input systems is recognised as an extremely difficult task. Indeed, for multiple-input systems, the quantisation density problem introduced in Elia and Mitter (2001) still remains largely open. Elia and Frazzoli (2002) and Elia (2002) provide lower bounds on the infimum quantisation density for two-input systems. Kao and Venkatesh (2002) analyse different quantisation schemes and their densities for linear multiple-input systems. However, explicit design of a multivariable quantiser with finite (though not necessarily infimum) quantisation density is performed only when quadratic stabilisation is possible through the use of a two-dimensional subspace of the input space.

The works of Fu and Xie (2003, 2005) employ a completely different approach to deal with the optimisation of quantisation density. These authors model a logarithmic quantiser as a nonlinearity

bounded by a sector. The system is then regarded as an uncertain system with sector-bound uncertainty and the problem is posed as a robust control problem. Their main finding is that, for single-input systems, this approach is not conservative. That is, the results of Elia and Mitter (2001) can be recovered by means of this approach. To deal with multiple-input systems, Fu and Xie utilise independent scalar quantisers for each input signal. Independently quantising the different input signals, however, leads to designs with infinite quantisation density.

Our contribution to this problem will be to give several new results related to quadratic stabilisation of single- and multiple-input systems and to quantisation density. These results are presented in Part I of the thesis and include the derivation of the structure of optimal quantisers and the explicit design of multivariable quantisers with finite density that are able to quadratically stabilise systems having an arbitrary number of inputs.

1.1.2 Componentwise Ultimate Bounds for Perturbed Systems

In Part II of the thesis, we deal with the derivation of componentwise ultimate bounds for continuous-time, discrete-time and sampled-data perturbed systems, especially when the perturbations arise due to the use of quantisers.

Quantisation in digital control systems arises due to the use of A/D, D/A and digital processors with finite precision. Since a digital control system is usually designed ignoring the presence of quantisation, it is necessary to estimate the effect that quantisation has on the resulting practical implementation. This effect can be quantified by means of bounds on the difference between the desired and the actual system behaviour. In particular, it is of interest to obtain ultimate bounds on the system variables (Yakowitz and Parker, 1973; Green and Turner, 1988; Miller et al., 1988; Farrell and Michel, 1989; Miller et al., 1989).

More recently, motivated by networked control systems, several control schemes have been considered that involve static memoryless quantisers (see for example, Elia and Mitter, 2001; Ishii and Francis, 2002b, 2003; Ishii et al., 2004; Ishii and Başar, 2005; Fu and Hara, 2005). Most of these works deal with the design of quantised control strategies to achieve different objectives, and utilise all the information provided by a quantised variable. However, some aspects of the resulting schemes can also be analysed by regarding a quantised variable as a perturbed copy of the corresponding unquantised variable. In particular, ultimate bounds on the system variables may be obtained in this manner when asymptotic stability is not possible.

Regarding a quantised variable as a perturbed copy of the unquantised variable turns a quantised control system into a perturbed system. The most general and powerful tool to analyse ultimate bounds in perturbed systems is the use of Lyapunov functions (see, for example, Khalil, 2002). This approach has the inherent difficulty of finding a suitable Lyapunov function. For linear systems, however, quadratic Lyapunov functions can be easily computed. Kofman (2005) proposes a different method to estimate

ultimate bounds for linear continuous-time perturbed systems with constant perturbation bounds. The method is based on the analysis of the system in modal coordinates and gives componentwise ultimate bounds on the system state. An example is given where the suggested method yields ultimate bounds that are substantially tighter than those derived by means of quadratic Lyapunov functions. The method of Kofman can be regarded as the continuous-time counterpart to the earlier method of Yakowitz and Parker (1973).

In Part II of the thesis, we will bring together the above earlier ideas and more recent work. In particular, motivated by the results of Yakowitz and Parker (1973) and Kofman (2005), we develop systematic methods to obtain componentwise ultimate bounds in continuous-time, discrete-time and sampled-data perturbed systems, especially when the perturbations arise due to the use of quantisers. We allow for different types of quantisers: uniform, logarithmic and semitruncated logarithmic. Our developments require several extensions of the methods of Yakowitz and Parker and Kofman. We also show how our methods can be applied to a class of nonlinear systems. The main features of our methods are their systematic nature and their flexibility in dealing with highly structured perturbation schemes. This latter feature allows us to deal with systems involving many different quantisers in the same setting.

1.2 Thesis Overview

The contents of the thesis are presented in 6 core chapters, which have been organised into two parts: the first part deals with stabilisation by means of quantisers, and the second part addresses the derivation of ultimate bounds in the presence of quantisation. A final chapter presents a summary and conclusions.

Part I (Chapters 2 to 5) addresses quantisation density in the context of quadratic stabilisation of discrete-time systems by means of static feedback utilising quantisers. The works most related to this part of the thesis are Elia and Mitter (2001); Elia and Frazzoli (2002); Elia (2002); Kao and Venkatesh (2002); Fu and Xie (2003, 2005). A more detailed description of the various chapters follows.

In Chapter 2, we first briefly review quadratic stabilisation of linear discrete-time systems and then focus on quantisation density in the context of multiple-input systems. We generalise the definition of quantisation density of Elia and Mitter (2001) to multiple-input systems and derive several new results regarding quantisation density. We also pose the problem of optimising quantisation density over all quantisers that quadratically stabilise a given multiple-input system and derive an important result that reveals the structure of a quantiser that optimises density. The different results of this chapter are employed in the remaining chapters of Part I of the thesis.

In Chapter 3, we focus on the characterisation of quantisers that quadratically stabilise a given multiple-input system. As a first step toward this characterisation, we consider quantisers having a form that can be interpreted as the simplest possible in some appropriate sense. We derive necessary and sufficient conditions for these quantisers to quadratically stabilise a system, and we do this by

means of explicit geometric considerations. We thus develop a novel geometric approach to quadratic stabilisation of multiple-input systems by means of quantisers. The geometric approach derived in this chapter will provide the framework for results derived in subsequent chapters. We also employ this geometric approach to design quantisers with finite density that can stabilise multiple-input systems having an arbitrary number of inputs.

In Chapter 4, we deal with single-input systems. For these systems, we enhance the geometric approach of Chapter 3 to explore quantiser coarseness from a state-space standpoint, as opposed to the standard input-space-based concept of quantisation density. We introduce a novel type of quantisers, namely CAQS (Coarse-Almost-Quadratically-Stabilising) quantisers, and analyse the relationships between CAQS quantisers and quantisers that minimise quantisation density in the standard sense. We also show how to directly utilise CAQS quantisers to design static output feedback strategies that employ quantisers of infimum density. We conclude this chapter by showing how to recover a well-known result on infimum quantisation density by means of our approach.

In Chapter 5, we solve a special case of infimum quantisation density problem for multiple-input systems. Specifically, we derive the infimum density over all quantisers that quadratically stabilise the system and have levels in a one-dimensional subspace of the input space. We also show that our result conflicts with a previously published result, and we provide a counterexample to the latter result.

Part II (Chapters 6 and 7) addresses the derivation of ultimate bounds in systems involving quantisation. Throughout this part, a quantised variable will be regarded as a perturbed copy of the corresponding unquantised variable.

In Chapter 6, we derive componentwise ultimate bound expressions for discrete-time and sampled-data perturbed systems, especially when the perturbations arise due to quantisation. A very important feature of our results is that they can directly accommodate feedback schemes where quantisers of different characteristics and/or types affect different signals in the same system. We demonstrate the applicability and potential of the method by means of an example taken from recent literature on the topic of control over communication networks.

In Chapter 7, we extend the results of Chapter 6 to deal with perturbed systems where the perturbation bounds have more general forms. In this case, we focus on continuous- and discrete-time perturbed systems. Since the perturbations are allowed to be bounded by state-dependent functions, the method can then be applied to nonlinear systems by regarding them as perturbed linear systems.

In Chapter 8, we summarize and give suggestions for future work.

1.3   Thesis Contributions

The main contributions of the thesis are believed to be:

Chapter 2. We derive an expression for the quantisation density of multivariable quantisers having radially logarithmically spaced levels (Theorem 2.12). We establish the invariance of the density of a quantiser under a linear one-to-one transformation (Lemma 2.15). We also derive results (Theorem 2.17 and Theorem 2.19) that provide insight into the structure of quantisers that optimise density for a multiple-input system.

Chapter 3. We give a geometric interpretation to the fact that a quantiser quadratically stabilises the system. We derive necessary and sufficient conditions for a quantised feedback having levels in a minimum-dimensional subspace to quadratically stabilise a given multiple-input system (Theorem 3.14 and Theorem 3.17). We also explicitly design quantisers having finite quantisation density that are able to quadratically stabilise a system having an arbitrary number of inputs (Theorem 3.22).

Chapter 4. For single-input systems, we explore quantiser coarseness from a state-space standpoint. We introduce and characterise CAQS quantisers (Theorem 4.16) and analyse the connections between this state-space approach and the standard input-space-based quantisation density (Theorems 4.19, 4.21, 4.22 and 4.23). We also show how to directly utilise CAQS quantisers to design static output feedback strategies that employ infimum density quantisers (Theorem 4.20).

Chapter 5. We derive a new result on infimum quantisation density for multiple-input systems, optimising over the class of quantisers that have levels in a one-dimensional subspace of the input space (Theorem 5.3). We also show that our result partially replaces an incorrect intermediate result in Elia and Frazzoli (2002).

Chapter 6. We derive ultimate bound expressions for perturbed discrete-time systems (Theorem 6.5) and sampled-data systems (Theorem 6.8 and Lemma 6.9). These expressions are believed to be novel. A key feature that distinguishes these results from other ultimate-bound derivation methods is the particular componentwise form of the perturbation bound [see (6.36)]. This form for the perturbation bound is particularly well-suited to the analysis of schemes where different combinations of uniform, logarithmic and semitruncated logarithmic quantisers are simultaneously employed on the same system.

Chapter 7. We extend the results of Chapter 6 to perturbed systems with more general componentwise perturbation bounds. This extension allows us to derive ultimate bounds for a class of nonlinear systems by regarding them as perturbed linear systems, with perturbation bounds that may depend on the system state. We provide systematic methods for the derivation of ultimate bounds (Theorem 7.4 for continuous-time systems and Theorem 7.8 for discrete-time systems), jointly with a region of attraction to the ultimate bound (Algorithm 1 and Theorem 7.5 for continuous-time systems, and Theorem 7.9 for discrete-time systems).

1.4 Associated Publications and Related Work

Most of the results presented in this thesis have been published by the author in journal and conference papers. The following list details the relevant publications:

Journal Papers

- H. Haimovich, M. M. Seron and G. C. Goodwin. Geometric Characterization of Multivariable Quadratically Stabilizing Quantizers. *International Journal of Control*, 79(8):845-857, 2006.

- E. Kofman, H. Haimovich and M. M. Seron. A Systematic Method to Obtain Ultimate Bounds for Perturbed Systems. *International Journal of Control*, 2006. In press.

Conference Papers

- H. Haimovich and M. M. Seron. On infimum quantization density for multiple-input systems. In *Proc. 44th IEEE Conf. on Decision and Control, Seville, Spain*, pp. 7692-7697, 2005.

- H. Haimovich. Stabilizing static output feedback via coarsest quantizers. In *16th IFAC World Congress, Prague, Czech Republic*, 2005.

Other related works published by the author during his Ph.D. studies are:

Book Chapters

- T. Perez and H. Haimovich. Output Feedback Optimal Control with Constraints. In G. C. Goodwin, M. M. Seron and J. A. De Doná, *Constrained Control and Estimation: An Optimisation Approach*, Springer-Verlag, London, 2005, Chapter 12.

- J. Welsh, H. Haimovich and D. Quevedo. Control over Communication Networks. In G. C. Goodwin, M. M. Seron and J. A. De Doná, *Constrained Control and Estimation: An Optimisation Approach*, Springer-Verlag, London, 2005, Chapter 16.

Journal Papers

- G. C. Goodwin, H. Haimovich, D. E. Quevedo and J. S. Welsh. A moving horizon approach to networked control system design. *IEEE Trans. on Automatic Control*, 49(9):1427-1445, 2004.

- T. Perez, H. Haimovich and G. C. Goodwin. On optimal control of constrained linear systems with imperfect state information and stochastic disturbances. *International Journal of Robust and Nonlinear Control*, 14:379-393, 2004.

Conference Papers

- H. Haimovich, G. C. Goodwin and J. S. Welsh. Set-valued observers for Constrained State Estimation of Discrete-time Systems with Quantized Measurements. In *Proc. 5th Asian Control Conference*, Melbourne, Australia, pp. 1947-1955, 2004.

- H. Haimovich, G. C. Goodwin and D. E. Quevedo. Moving horizon Monte Carlo state estimation for linear systems with output quantization. In *Proc. 42nd IEEE Conf. on Decision and Control, Maui, HI, USA*, pp. 4859-4864, 2003.

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- H. Haimovich, M. M. Seron, G. C. Goodwin and J. C. Agüero. A neural approximation to the explicit solution of constrained linear MPC. In *Proc. European Control Conference*, Cambridge, UK, 2003.

Confidential Reports for Industry

- J.S. Welsh, G.C. Goodwin, H. Haimovich, H. Tidefelt, A. Rosen and R.H. Middleton, *'Adaptive Powertrain Control: Report 4 - Final Report'*, Report for General Motors, USA, January, 2004.

- A. Rosen, G.C. Goodwin, J.S. Welsh, R.H. Middleton and H. Haimovich, *'Adaptive Powertrain Control: Interim Report No. 5'*, Report for General Motors, USA, December, 2003.

- H. Tidefelt, G.C. Goodwin, J.S. Welsh, R.H. Middleton and H. Haimovich, *'Adaptive Powertrain Control: Interim Report No. 4'*, Report for General Motors, USA, October, 2003.

- J.S. Welsh, H. Haimovich, G.C. Goodwin, R.H. Middleton and J.A. De Dona, *'Adaptive Powertrain Control: Interim Report No. 3'*, Report for General Motors, USA, July, 2003.

- J.S. Welsh, H. Haimovich, G.C. Goodwin, R.H. Middleton and J.A. De Dona, *'Adaptive Powertrain Control: Interim Report No. 2'*, Report for General Motors, USA, June, 2003.

- J.S. Welsh, H. Haimovich, G.C. Goodwin, R.H. Middleton and J.A. De Dona, *'Adaptive Powertrain Control: Interim Report No. 1'*, Report for General Motors, USA, May, 2003.

- J.S. Welsh, H. Haimovich, G.C. Goodwin, R.H. Middleton and J.A. De Dona, *'Adaptive Powertrain Control: Report 3 - System Identification for Automotive Powertrain Control'*, Report for General Motors, USA, March, 2003.

- J.S. Welsh, H. Haimovich, G.C. Goodwin, R.H. Middleton, J.C. Agüero, J.A. De Dona and M.M. Seron, *'Adaptive Powertrain Control Report No.2a'*, Report for General Motors, USA, November, 2002.